Radioactivity and Half-life

Now let's look at **radioactivity and half-life**. The half-life of a radioactive element is **the time it takes for half of the radioactive nuclei present in a sample to decay**. It is a curious thing, since it makes no difference how much of the element is in the sample. If the half life of an element is 10 days and we have a sample of 100 g, then in 10 days we will have 50 g (half of the sample is gone), in another 10 days there will be 25 g left (half of the 50 g is gone), and so on. Every 10 days half of the remaining radioactive nuclei will decay, no matter how big that "half" is! It is mind-boggling.

This is the concept on which **Carbon-14 dating** is based. The amount of radioactive carbon in a living organism stays constant during its lifetime because it continually ingests replacement carbon-14. However, after its death the amount of carbon-14 will begin to decrease as the carbon decays. The half life of carbon-14 is about 5700 years, so by comparing the amount of carbon-14 present in a sample with the amount assumed to be in the original organism we can get an estimate of the age of the sample.

When we do these types of problems, we use the **Law of Exponential Change**:

$$y = y_0 e^{kt}$$

When k > 0, the material is growing (like populations of cities or cases of a disease). When k < 0, the material is shrinking (like radioactive material in a sample).

Here is a problem where we are supposed to find out how old Crater Lake is based on the charcoal left in a tree killed in the volcanic eruption. This charcoal sample contains 44.5% of the carbon-14 found in living matter.

$0.5 y_0 = y_0 e^{5700k}$ $0.5 = e^{5700k}$	First we will use the half-life (5700 years) to find k. We know that half of the original amount is left after 5700 years.
$\ln 0.5 = 5700k$ $k = \dfrac{\ln 0.5}{5700}$	Write this in log form and solve for k. **It is best to use the exact expression for k and not a rounded decimal or you will end up with an inaccurate answer when you get to the end of the problem.** And the AP will not give credit for that type of rounding error!
$0.445 y_0 = y_0 e^{k*t}$ $0.445 = e^{k*t}$ $\ln 0.445 = k*t$ $t = \dfrac{\ln 0.445}{k} = \dfrac{5700(\ln 0.445)}{\ln 0.5}$ $t \approx 6658$ years old	Now we put this back into the equation and find t when $y = 0.445\, y_0$ (44.5% of the original amount is present). **Notice that I just used "k"** instead of writing that fraction out. Store your value of k in the calculator or just put it in at the last step when you need an answer. Write this in log form and solve for t. Now we know that Crater Lake is approximately 6658 years old.

Note:
If you look at the book, you will see a formula for half-life. Please, do not memorize it! It is not worth the space in your brain! You have to learn the basic growth and decay formula,

$y = y_0 e^{kt}$, anyway. And for half-life, we just have to realize that half-life means that only half of what you started with is still left: $\frac{1}{2} y_0 = y_0 e^{kt}$ which simplifies into $\frac{1}{2} = e^{kt}$. Bingo, you have what you need without memorizing another formula.